

Matching the quark model to the $1/N_c$ expansion

Dan Pirjol* and Carlos Schat[†]

* *National Institute for Physics and Nuclear Engineering, Department of Particle Physics,
077125 Bucharest, Romania*

[†] *Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701, USA
Departamento de Física, FCEyN, Universidad de Buenos Aires, Ciudad Universitaria, Pab.1, (1428) Buenos
Aires, Argentina*

Abstract. We compute the coefficients of the effective mass operator of the $1/N_c$ expansion for negative parity $L = 1$ excited baryons using the Isgur-Karl model in order to compare the general approach, where the coefficients are obtained by fitting to data, with a specific constituent quark model calculation. We discuss the physics behind the fitted coefficients for the scalar part of the most general two-body quark-quark interaction. We find that both pion exchange and gluon exchange lead to the dominance of the same operator at the level of the effective mass operator, which is also observed from data.

Keywords: Quark model, $1/N$ expansion of QCD, excited baryons

PACS: 12.39.-x, 12.38.-t, 14.20.-c

INTRODUCTION

Quark models provide a simple picture of the physics of ground state baryons and their excitations [1, 2], but their connection with QCD is not so clear. An alternative approach to baryon phenomenology that is closer connected to QCD is provided by the $1/N_c$ expansion [3], which has been applied to the study of both the ground state and excited nucleons [4, 5, 6, 7, 8, 9, 10], leading to interesting insights that would be difficult to obtain in a particular model calculation.

In order to bridge the gap between the more intuitive quark model calculations and the effective operator approach, in a recent paper [11] we showed how to match an arbitrary quark model Hamiltonian onto the operators of the $1/N_c$ expansion. We made use of the transformation of the states and operators under the permutation group of N_c objects acting on the spin-flavor of the quarks. The approach is similar to the one discussed in Ref. [12] for the orbital part of the interaction and at $N_c = 3$. The main result of [11] can be summarized as follows: consider a two-body quark Hamiltonian $V_{qq} = \sum_{i<j} O_{ij} R_{ij}$, where O_{ij} acts on the spin-flavor quark degrees of freedom and R_{ij} acts on the orbital degrees of freedom. Then the hadronic matrix elements of the quark Hamiltonian on a baryon state $|B\rangle$ contains only the projections O_α onto irreducible representations of the spin-flavor permutations and can be factorized as $\langle B|V_{qq}|B\rangle = \sum_\alpha C_\alpha \langle O_\alpha \rangle$, where the coefficients C_α are reduced matrix elements of the orbital operators R_{ij} , given by overlap integrals of the quark model wave functions.

The matching procedure has been discussed in detail for the Isgur-Karl (IK) model in Ref. [13]. As an application of the general approach in Ref. [14, 15] we examine the spectrum of $L = 1$ negative parity baryons and discuss the implications for the spin-flavor structure of the most general two-body quark interaction. The present talk summarizes some aspects of this recent work that strive to understand the physics hidden in the numerical values of the fitted coefficients of the general operator expansion.

THE MASS OPERATOR OF THE ISGUR-KARL MODEL

The Isgur-Karl model is defined by the quark Hamiltonian

$$\mathcal{H}_{IK} = \mathcal{H}_0 + \mathcal{H}_{\text{hyp}}, \quad (1)$$

¹ Speaker

where \mathcal{H}_0 contains the confining potential and kinetic terms of the quark fields, and is symmetric under spin and isospin. The hyperfine interaction \mathcal{H}_{hyp} is given by

$$\mathcal{H}_{\text{hyp}} = A \sum_{i < j} \left[\frac{8\pi}{3} \vec{s}_i \cdot \vec{s}_j \delta^{(3)}(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} (3\vec{s}_i \cdot \hat{r}_{ij} \vec{s}_j \cdot \hat{r}_{ij} - \vec{s}_i \cdot \vec{s}_j) \right], \quad (2)$$

where A determines the strength of the interaction, and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ is the distance between quarks i, j . The first term is a local spin-spin interaction, and the second describes a tensor interaction between two dipoles. This interaction Hamiltonian is an approximation to the gluon-exchange interaction, neglecting the spin-orbit terms.

In the original formulation of the IK model [2] the confining forces are harmonic. We will derive in the following the form of the mass operator without making any assumption on the shape of the confining quark forces. We refer to this more general version of the model as IK-V(r).

The $L = 1$ quark model states include the following SU(3) multiplets: two spin-1/2 octets $8_{\frac{1}{2}}, 8'_{\frac{1}{2}}$, two spin-3/2 octets $8_{\frac{3}{2}}, 8'_{\frac{3}{2}}$, one spin-5/2 octet $8'_{\frac{5}{2}}$, two decuplets $10_{\frac{1}{2}}, 10_{\frac{3}{2}}$ and two singlets $1_{\frac{1}{2}}, 1_{\frac{3}{2}}$. States with the same quantum numbers mix. For the $J = 1/2$ states we define the relevant mixing angle θ_{N1} in the nonstrange sector as $N(1535) = \cos \theta_{N1} N_{1/2} + \sin \theta_{N1} N'_{1/2}$, $N(1650) = -\sin \theta_{N1} N_{1/2} + \cos \theta_{N1} N'_{1/2}$ and similar equations for the $J = 3/2$ states, which define a second mixing angle θ_{N3} .

We find [13] that the most general mass operator in the IK-V(r) model depends only on three unknown orbital overlap integrals, plus an additive constant c_0 related to the matrix element of \mathcal{H}_0 , and can be written as

$$\hat{M} = c_0 \mathbf{1} + a S_c^2 + b L_2^{ab} \{S_c^a, S_c^b\} + c L_2^{ab} \{s_1^a, s_c^b\}, \quad (3)$$

where the spin-flavor operators are understood to act on the state $|\Phi(SI)\rangle$ constructed as a tensor product of the core of quarks 2,3 and the ‘excited’ quark 1, as given in [6, 11]. The coefficients are given by $a = \frac{1}{2} \langle R_S \rangle$, $b = \frac{1}{12} \langle Q_S \rangle - \frac{1}{6} \langle Q_{MS} \rangle$, $c = \frac{1}{6} \langle Q_S \rangle + \frac{1}{6} \langle Q_{MS} \rangle$. The reduced matrix elements R_S, Q_S, Q_{MS} for the orbital part of the interaction contain the unknown spatial dependence and are defined in Refs. [6, 13]. Evaluating the matrix elements using Tables II, III in Ref. [6] we find the following explicit result for the mass matrix

$$M_{1/2} = \begin{pmatrix} c_0 + a & -\frac{5}{3}b + \frac{5}{6}c \\ -\frac{5}{3}b + \frac{5}{6}c & c_0 + 2a + \frac{5}{3}(b+c) \end{pmatrix}, \quad M_{3/2} = \begin{pmatrix} c_0 + a & \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c \\ \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c & c_0 + 2a - \frac{4}{3}(b+c) \end{pmatrix}, \quad (4)$$

and $M_{5/2} = c_0 + 2a + \frac{1}{3}(b+c)$, $\Delta_{1/2} = \Delta_{3/2} = c_0 + 2a$. Computing the reduced matrix elements with the interaction given by Eq. (2), one finds that the reduced matrix elements in the IK model with harmonic oscillator wave functions are all related and can be expressed in terms of the single parameter δ as $\langle Q_{MS} \rangle = \langle Q_S \rangle = -\frac{3}{5}\delta$; $\langle R_S \rangle = \delta$. This gives a relation among the coefficients a, b, c of the mass matrix Eq. (3): $a = \frac{1}{2}\delta$, $b = \frac{1}{20}\delta$, $c = -\frac{1}{5}\delta$. We recover the well known result that in the harmonic oscillator model, the entire spectroscopy of the $L = 1$ baryons is fixed by one single constant $\delta = M_\Delta - M_N \sim 300$ MeV, along with an overall additive constant c_0 , and the model becomes very predictive. In Fig. 1 we show the result of a best fit of a, b, c in the IK-V(r) model together with the predictions of the IK model. The IK-V(r) spectrum is the best fit possible for a potential model with the spin-flavor interaction given in Eq. (2).

TESTING A MORE GENERAL SPIN-FLAVOR STRUCTURE

The most general quark Hamiltonian containing only two-body interactions has the form [14]

$$H_{qq} = H_0 + \sum_{i < j} (f_1(\vec{r}_{ij}) t_i^a t_j^a + f_2(\vec{r}_{ij}) \vec{s}_i \cdot \vec{s}_j + f_3(\vec{r}_{ij}) \vec{s}_i t_i^a \cdot \vec{s}_j t_j^a) + H_{s-o} + H_q, \quad (5)$$

where H_0 is the part of the quark Hamiltonian which does not depend on the quarks spin and flavor degrees of freedom. We show explicitly only the part of the Hamiltonian which transforms as a scalar ($\ell = 0$) under $SO(3)$, the group of orbital rotations - the scalar part of the quark Hamiltonian. The H_{s-o}, H_q denote the spin-orbit and the quadrupole interaction, which transform as a vector ($\ell = 1$) and a tensor ($\ell = 2$), respectively.

We will consider the case of contact scalar interactions

$$f_v(\vec{r}_{ij}) = A_v \delta^{(3)}(\vec{r}_{ij}), \quad v = 1, 2, 3, \quad (6)$$

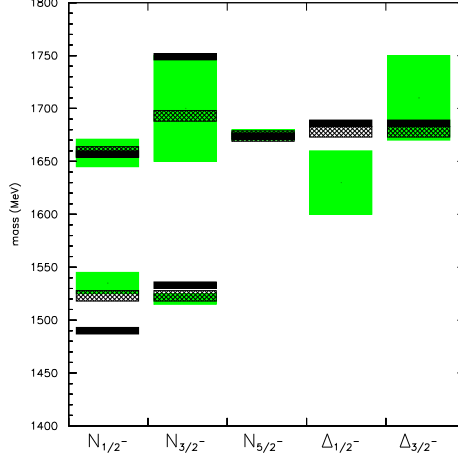


FIGURE 1. Masses predicted by the IK model (black bars), by the IK-V(r) model (hatched bars) and the experimental masses (green boxes) from Ref. [18].

and study the following question: what information can be obtained from the coefficients c_k of the $1/N_c$ studies of the spectrum of $L = 1$ negative parity baryons?

The quark Hamiltonian H_{qq} is matched onto the hadronic mass operator

$$M = c_0 \mathbf{1} + c_1 T^2 + c_2 S_c^2 + c_3 \vec{s}_1 \cdot \vec{S}_c + \dots \quad (7)$$

where the ellipses denote terms arising from the tensor and spin-orbit interactions, which were considered in Ref. [14]. All three possible zero range two-body interactions $O_{ij}^{(v)} = t_i^a t_j^a, \vec{s}_i \cdot \vec{s}_j$ and $\vec{s}_i t_j^a \cdot \vec{s}_j t_j^a$ lead to $c_1 = c_3 = 0$ and are matched onto the same operator $O_2 = S_c^2$ in the effective theory. This means that there is no way to distinguish between these three types of scalar interactions as long as they are of very short range. Experimentally, at $N_c = 3$ one can determine only two linear combinations of the three coefficients $c_{1,2,3}$ (as functions of $\theta_{N1,N3}$) [14] from the mass spectrum and mixing angles of the negative parity $L = 1$ baryons. These linear combinations can be taken as $\tilde{c}_1 = c_1 - \frac{1}{2}c_3, \tilde{c}_2 = c_2 + c_3$ and their explicit expressions in terms of the nonstrange hadron masses and mixing angles can be found in Ref. [15].

A first estimate of $\tilde{c}_{1,2}$ can be made using the mixing angles $\theta_{N1,N3}$ determined from a fit to N^* strong decays and photoproduction data $(\theta_{N1}, \theta_{N3}) = (0.39 \pm 0.11, 2.82 \pm 0.11) = (22^\circ \pm 6^\circ, 162^\circ \pm 6^\circ)$ [16, 17]. Using these values and the hadron masses from the PDG [18] we obtain $\tilde{c}_1 = 3.9 \pm 11.0$ MeV, $\tilde{c}_2 = 129 \pm 18$ MeV. The corresponding ratio of the coefficients \tilde{c}_i displays the relative suppression of \tilde{c}_1

$$\tilde{c}_1/\tilde{c}_2 = 0.03 \pm 0.09. \quad (8)$$

An alternative determination of this ratio can be made using only the excited baryon masses, as discussed in Ref. [14]. In this paper it was shown that in any quark model containing only two-body quark interactions there is a correlation between both mixing angles (up to a discrete ambiguity) that corresponds to the dashed and solid lines in Fig. 2. The scatter plot takes into account the experimental errors in the determination of the baryon masses. A second relation (Eq. (6) in Ref. [14]) expressing the spin-average of the $SU(3)$ singlet states $\bar{\Lambda} = \frac{1}{3}\Lambda_{1/2} + \frac{2}{3}\Lambda_{3/2}$ in terms of the nonstrange states further constrains the mixing angle and ratio to the dark (green) area in the plot and singles out the solid line as the preferred solution. In Fig. 2 the first determination of the ratio Eq. (8) is shown as the black point with error bars and falls within the dark (green) area.

Finally, it is interesting to notice [6, 15] that for the particular case ($v = 3$) of the spin-flavor structure corresponding to one-pion exchange \tilde{c}_1 vanishes regardless of the spatial range of the orbital part of the interaction (although for non-zero range $c_{1,3}$ do not vanish separately).

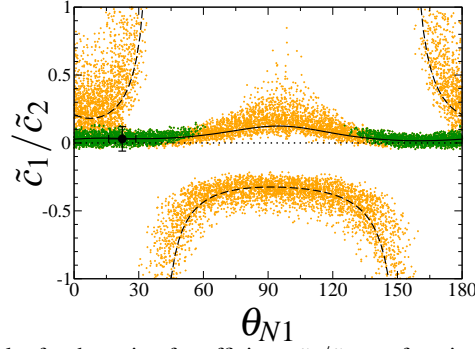


FIGURE 2. Scatter plot for the ratio of coefficients \tilde{c}_1/\tilde{c}_2 as a function of the mixing angle θ_{N1} .

CONCLUSIONS

We presented the effective mass operator of the Isgur-Karl model in the operator basis used in the $1/N_c$ expansion for excited baryons. This explicit model calculation of the operator coefficients (for details see Ref. [13]) connects two different approaches to baryon phenomenology and is an illustration of the general matching procedure discussed in Ref. [11] using the permutation group.

Considering the most general spin-flavor structure for the scalar part of the two-body quark interactions we find [15] that in the case of contact interactions all spin-flavor structures are matched onto the same effective operator in the $1/N_c$ expansion. For the particular case of a pion exchange interaction this holds independently of the range of the spatial part of the interaction. Comparing the values obtained for the operator coefficients from masses and independent determinations of mixing angles we find that a mixture of pion exchange and massless gluon exchange interactions is compatible with data, giving support to the physical picture considered in Refs. [19, 20].

ACKNOWLEDGMENTS

The work of C.S. was supported by CONICET and partially supported by the U. S. Department of Energy, Office of Nuclear Physics under contract No. DE-FG02-93ER40756 with Ohio University.

REFERENCES

1. A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).
2. N. Isgur and G. Karl, Phys. Lett. B **72**, 109 (1977).
3. R. F. Dashen, E. Jenkins and A. V. Manohar, Phys. Rev. D **49**, 4713 (1994) [Erratum-ibid. D **51**, 2489 (1995)]; R. F. Dashen, E. Jenkins and A. V. Manohar, Phys. Rev. D **51**, 3697 (1995).
4. J. L. Goity, Phys. Lett. B **414**, 140 (1997).
5. D. Pirjol and T. M. Yan, Phys. Rev. D **57**, 1449 (1998); Phys. Rev. D **57**, 5434 (1998).
6. C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Lett. B **438**, 327 (1998); Phys. Rev. D **59**, 114008 (1999).
7. C. L. Schat, J. L. Goity and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002); J. L. Goity, C. L. Schat and N. N. Scoccola, Phys. Rev. D **66**, 114014 (2002).
8. J. L. Goity, C. Schat and N. N. Scoccola, Phys. Lett. B **564**, 83 (2003).
9. D. Pirjol and C. Schat, Phys. Rev. D **67**, 096009 (2003); AIP Conf. Proc. **698**, 548 (2004).
10. N. Matagne and F. Stancu, Phys. Rev. D **71**, 014010 (2005); Phys. Lett. B **631**, 7 (2005); Phys. Rev. D **74**, 034014 (2006).
11. D. Pirjol and C. Schat, Phys. Rev. D **78**, 034026 (2008).
12. H. Collins and H. Georgi, Phys. Rev. D **59**, 094010 (1999).
13. L. Galetta, D. Pirjol and C. Schat, Phys. Rev. D **80**, 116004 (2009).
14. D. Pirjol and C. Schat, Phys. Rev. Lett. **102**, 152002 (2009).
15. D. Pirjol and C. Schat, arXiv:1007.0964 [hep-ph].
16. J. L. Goity, C. Schat and N. Scoccola, Phys. Rev. D **71**, 034016 (2005).
17. N. N. Scoccola, J. L. Goity and N. Matagne, Phys. Lett. B **663**, 222 (2008).
18. C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
19. A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).
20. S. Weinberg, arXiv:1009.1537 [hep-ph].